All solutions and teacher notes in blue

AP Statistics Handout Key: Lesson 2.1

Topics: percentiles, cumulative relative frequency, standardized scores (z-scores)

Lesson 2.1 Guided Notes

SAT: 1050

ACT: 23

Two Test Takers: Mr. Young-Saver (picture is an actual photo of him in high school) took the SAT and scored a 1050. Guy Fieri took the ACT and scored a 23. The SAT has a total of 1600 possible points. The ACT has a total of 36 possible points.

Who do you think had the better score? Explain your reasoning.

Answers will vary. One possible student answer:
1050/1600 = 66% | 23/36 = 64% . So, Mr. Young-Saver scored the highest proportion of possible points, so he had the better score

Percentiles

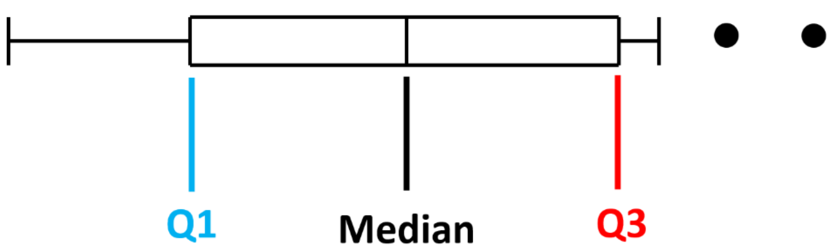
Percentile: the percent of data **less than or equal to** a certain data value.

Below are the yearly salaries of employees at a company (in thousands of $). At what percentile is the person who makes a salary of $43,000? Show your work.

29, 32, 34, 34, 34, 34, 35, 35, 39, 43, 67, 185

10/12 ≈ 83% → The salary of $43,000 is at the 83rd percentile of salaries.

For a large dataset…



1) What percentile is Q1?

Q1 is at the 25th percentile

2) What percentile is the median?

Q2 is at the 50th percentile

3) What percentile is Q3?

Q3 is at the 75th percentile

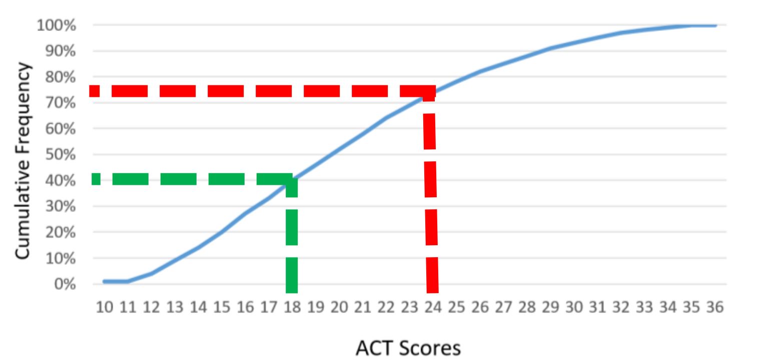
Back to our two test takers: These tables display the percentiles of different scores on the SAT and ACT (based on 2019-2020 exam year). Using this information, determine whose score (Mr. Young-Saver's or Guy Fieri's) is more impressive. Explain your reasoning.

Guy Fieri's score is relatively more impressive because he tied or outscored 69% of those who took the ACT, while Mr. Young-Saver only tied or outscored 45% of those who took the SAT.

SAT Score	Percentile
1600	100%
1550	99.3%
1500	98%
1450	95%
1400	93%
1350	89%
1300	84%
1250	78%
1200	71%
1150	62%
1100	53%
1050	45%
1000	35%
950	26%
900	19%
850	13%
800	8%
750	5%
700	3%
650	2%
600	1%

ACT Score	Percentile
36	100%
35	99.9%
34	99.0%
33	98%
32	97%
31	95%
30	93%
29	91%
28	88%
27	85%
26	82%
25	78%
24	74%
23	69%
22	64%
21	58%
20	52%
19	46%
18	40%
17	33%
16	27%
15	20%
14	14%
13	9%
12	4%
11	1%
10	1%

Cumulative Relative Frequency



Above is a cumulative relative frequency chart of the ACT data. Answer the following questions, using the chart:

1) Is 18 a good ACT score? Explain.

A score of 18 is at or above 40% of ACT test takers. So, it's an ok score.

2) You are applying for an elite college and want to score in the top quartile of test takers. What score do you need?

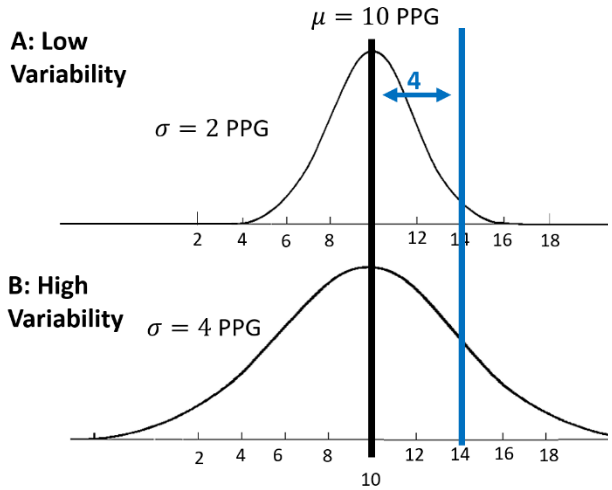
You would need a score of 24 (Q3) or higher to be in the top 25%

Standardized Scores (Z-Scores)

Z-Scores (also called standardized scores): measures how many **standard deviations** a data point is **above/below** the mean.

$$z = \frac{data\ point - mean}{standard\ deviation} \quad | \quad z = \frac{x_i - \mu}{\sigma}$$

Why Standard Deviation Matters:



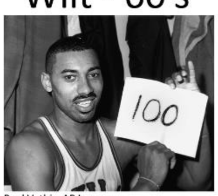
Here is another real photo of Mr. Young-Saver, playing basketball in high school. Doesn't he look like an elite athlete? Imagine that Mr. Y-S scores **14 points per game (PPG).**

In league A, players average 10 points per game, with a standard deviation of 2 points per game. Players in league B have the same average PPG, but with a higher standard deviation between them (4 PPG). In which league (A or B) is Mr. Young-Saver's performance more impressive? Why?

His performance is more impressive in League A because there is less variation in League A – therefore, there are fewer players who score more than him. Because he is more standard deviations above the mean in League A, he is more "unusually good" in that league.

Standardization: A point's location in the distribution depends on **both** distance from the **center** and the distribution's **spread or variation**.

Standardized: Who Was the Best?

	Wilt - 60's	Jordan - 90's	LeBron - 2010's
	 Paul Vathis, AP Images		
	30.1 ppg	30.1 ppg	27.1 ppg
Mean (μ) ppg in their era:	10.8 ppg	8.7 ppg	8.4 ppg
Stdv (σ) ppg in their era:	7.0 ppg	5.9 ppg	5.5 ppg
Z-Score: (show work)	$\frac{30.1-10.8}{7.0} = 2.8$	$\frac{30.1-8.7}{5.9} = 3.6$	$\frac{27.1-8.4}{5.5} = 3.4$

Q: Who was the **G.O.A.T?**

Explain your answer based on the z-scores you calculate:

Michael Jordan's PPG was **3.6** standard deviations **above** the mean for his era, making him the most **unusually high** scorer among these three legends.

Standardized: Players who were not the best…

Photo: opencourt-basketball.com

Adam Morrison

Calculate Adam Morrison's z-score for PPG…
While with the Lakers, he averaged **2.2** PPG.
(League: μ = 8.4 ppg, σ = 5.5 ppg)

$$z = \frac{\mathbf{2.2} - 8.4}{5.5} = \mathbf{-1.1}$$

Adam Morrison's scoring rate was 1.1 standard deviations **below** the league average in his era.

data value > mean → **positive**

data value < mean → **negative**

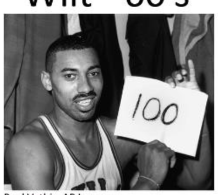
$$z = \frac{\overbrace{data\ point - mean}}{standard\ deviation}$$

Positive Z-Score: The number of standard deviations **above** the mean.

Negative Z-Score: The number of standard deviations **below** the mean.

Recommended discussion norms: skewthescript.org/discussion-norms

Lesson 2.1 Discussion

	Wilt - 60's	Jordan - 90's	LeBron - 2010's
	 Paul Vathis, AP Images		
Shots per game:	22.5	**22.9**	19.6
% shots made:	54.0%	**49.7%**	50.4%

Note: All data is from 2020 season and earlier

Discussion: Given these additional statistics, do you believe Jordan is still the G.O.A.T at scoring? Why or why not? What other stats may be helpful in determining who was the best?

Example arguments, against Michael Jordan:

- Scoring isn't just about points – it's about efficiency. Whenever you take a shot, you're taking away the opportunity for your teammate to take that shot. So, you better make your shots often. Even though Jordan got many points, he also shot and missed a lot (he had the lowest "make" percentage of the three players). The other players were more efficient.

Example arguments, in favor of Michael Jordan:

- The 3-point line wasn't added to the NBA until after Wilt retired. So, many players of his era (including Wilt) shot close to the rim. Because of this, they had high shooting percentages. Comparing shooting percentages across the 2 vs. 3 point eras may be flawed.

- LeBron plays alongside other superstars, so the defense is not solely focused on him. Jordan played with good teammates, but not many superstars. So, the defense focused mainly on him. Despite this, Jordan's shooting percent is less than 1% shy of LeBron's. That's impressive.

Teacher note: Ultimately, this sort of statistical question of "who was best" is somewhat subjective. Do you choose to look at total points? At efficiency? At strength of defense? At "clutch" moments? Expert sports statisticians debate these metrics. Ultimately, there is no single "correct" answer.

Lesson 2.1 Practice

Teachers: We recommend providing additional practice exercises from your AP Stats textbook or from prior AP Stats exams. The following textbook sections and AP exam questions are aligned to the content covered in this lesson.

- *The Practice of Statistics (AP Edition)*, 4th-6th editions: section 2.1
- *Stats: Modeling the World (AP Edition)*, 4th & 5th editions: ch 3 & 5, 3rd edition: ch 4 & 6
- *Statistics: Learning from Data (AP Edition)*, 2nd edition: section 3.5
- *Advanced High School Statistics*, section 2.2
- AP Exam Free Response Questions (FRQs): 2011 Q1 (parts b & c), 2019 Q6 (parts d & e)

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